ZENO'S PARADOXES

Douglas M. Beaumont
Charlotte, NC
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Zeno's Importance

Zeno of Elea (489 B.C.) was a disciple of Parmenides, himself a former Pythagorean who rejected their view in favor of Monism – the idea that all Being is one and that change is illusory.[1] Zeno stands as the first to attempt to counter a philosophical system rather than simply introduce a competing version. Sinnige says that Zeno's theories "opened the way for a new philosophical method, which was to be critical, in the sense that it could test the validity of its own presuppositions by using them in argument," and refers to Zeno as "the father of the dialectic."[2]

He is best known for his composition of a series of paradoxes – arguments he set forth to prove the position of Parmenides. He does this by assuming the opposing Pythagorean position (that reality is made up of discreet units), and shows several seemingly insoluble problems that result. This is *reductio ad absurdum* – the reduction of a conclusion to the absurd. Zeno set forth forty proofs which focus on the nature of reality and infinity. The most famous of these are: (1) *The Achilles*, (2) *The Dichotomy*, (3) *The Arrow*, and (4) *The Stadium*. The paradoxes of *The Achilles* and *The Dichotomy* are intended to show that space and time are not continuous by

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[1]Frederick Copleston, *A History of Philosophy vol. 1* (New York: Doubleday, 1993), 48-53.

[2]Theo Sinnige, *Matter and Infinity in the Presocratic Schools and Plato* (Assen, Netherlands: Koninklijke Van Gorcum & Comp., 1968), 98.

demonstrating the impossibility of motion, the paradoxes of *The Arrow* and *The Stadium* are designed to show that the atomistic view of the Pythagoreans is false. This paper will focus on the former, those dealing with the absurdity of movement through an infinite series of points.

Although some of the arguments may sound comical at first hearing, one should approach with respect. Zeno's impact spans from Aristotle's time to our own. They continue to interest not only the philosophers, but also the mathematicians and physicists of today. Wesley Salmon begins his writing on Zeno's paradoxes with a warning to anyone thinking that they are "mere anachronisms that . . . only show how mathematically naive were the Greeks of the fifth century B.C." by stating, "No evaluation could be further from the truth."[3] Bertrand Russell commented, "Zeno's arguments, in some form, have afforded grounds for almost all theories of space and time and infinity which have been construed from his time to our own."[4] While entertaining, these paradoxes are not "simply witty toys, but are calculated to prove the position of his master."[5] In the end, these arguments may not prove Parmendean Monism, they at least makes it look less ridiculous than it might first appear: "[Parmenides'] opponents attempted to ridicule the teaching that all things are one. Zeno's tactic was to show how the tenet that things are multiple turns out to be even more ridiculous."[6] The significance of this possibility is obvious: "in Zeno's

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[3]Wesley Salmon, *Zeno's Paradoxes*, (New York: The Bobbs-Merrill Company, Inc., 1970), v.

[4]Bertrand Russel, *The Problem of Infinity Considered Historically*, in *Zeno's Paradoxes* ed. Wesley Salmon (New York: The Bobbs-Merrill Company, Inc., 1970), 54.

[5]Ibid., 54.

[6]Joseph Owens, *A History of Ancient Western Philosophy* (New York: Appleton-Century-Crofts, Inc., 1959), 80.

paradoxes infinity plays the role of a dissolvent of reality."[7] Below will be presented two of Zeno's most famous paradoxes, along with their proposed solutions and implications.

Achilles and the Tortoise

Lacking Zeno's original writings we are reliant upon Aristotle for their descriptions. He recounts this paradox as follows: "The second [argument] is the so-called 'Achilles', and it amounts to this, that in a race the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead."[8] Salmon gives a more robust description:

> Imagine that Achilles, the fleetest of Greek warriors, is to run a footrace against a tortoise. It is only fair to give the tortoise a head start. . . . In order to overtake the tortoise, Achilles must run from his starting point to the tortoise's original starting point T_0. While he is doing that, the tortoise will have moved ahead to T_1. . . . And so it continues; whenever Achilles arrives at a point where the tortoise *was*, the tortoise has already moved a bit ahead [see fig. 1]. Achilles can narrow the gap between him and the tortoise, but he can never actually catch up with him.[9]

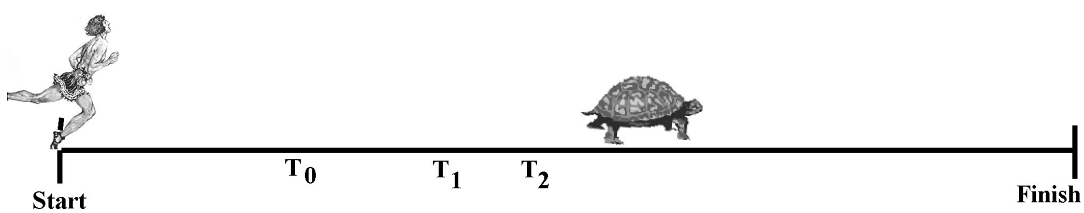

Figure 1.

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[7]N.Y. Vilenkin, *In Search of Infinity*, trans. Abe Shenitzer (Boston: Birkhauser, 1995), 8.

[8]Aristotle, *Physics,* Book VI:9 239b in *The Great Books of the Western World vol.8 Aristotle: 1*, ed. Robert Hutchins, trans. W.D. Ross (Chicago: William Benton - Encyclopedia Britannica, 1952), 323.

[9]Salmon, 8-9.

It may seem easy enough to show this paradox to be false relying merely on intuition. But this paradox cannot be resolved by demonstration.[10] Of course even Zeno realizes that in an actual footrace Achilles would win, or at least *appear* to win. But this is exactly Zeno's point: the appearance is false. To solve the paradox one must "show either that the inference is invalid or that the premiss is false. . . . what matters is not to prove that Achilles must reach the tortoise but to find where Zeno's argument breaks down."[11] The next paradox takes this idea a step further.[12]

The Dichotomy

The second paradox is similar to *Achilles*, but builds upon it. Zeno claims that, tortoise or not, Achilles could not even *finish* the race. The reason for this is that for Achilles to reach the finish line he must first pass through the point which is half of the distance. In order to pass through this point, however, Achilles must pass the point half way to that point (see fig. 2). These divisions are without end, therefore Achilles cannot finish the race.

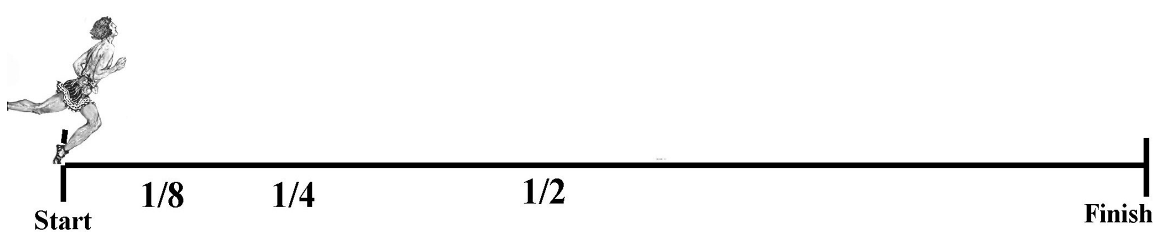

Figure 2.

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[10]Upon hearing Zeno's arguments Diogenes simply got up and walked!

[11]J.O. Wisdom, *Achilles on a Physical Racecourse* in Salmon, *Zeno's Paradoxes*, (New York: The Bobbs-Merrill Company, Inc., 1970), 83.

[12]Pun intended.

Here Zeno has not only shown that Achilles cannot finish the race, but that he really cannot even begin to run it (for even the first step is divisible). Once again, in our perception of the race Zeno is mistaken. But if this is the case then there must be a flaw in Zeno's thinking and if so it should be able to be discovered without resorting to empirical proofs which he would only deny. In the attempt to do so one must first identify the specific argument in a form that can be more easily dissected. Below is presented the basic structure of these paradoxes.

Explanation of the Essential Problem

Both *Achilles* and *Dichotomy* are established upon one basic problem: the impossibility of accomplishing an infinite number of actions (traversing an infinite number of points) in a finite amount of time. In syllogistic form it could be rendered:

(1) In order for movement to take place an infinite number of points must be traversed.
(2) An infinite number of points cannot be traversed.
(3) Therefore movement cannot take place.

For any valid syllogism of this type the only way to avoid the conclusion is to show either that it is invalid in form, or that one or both of its premisses are false. The task at hand is summed up well by James Thomson when he writes,

> It may seem that this argument is valid; and then, since the conclusion is absurd, we must deny one of the premisses. But which? Each has a certain plausibility. To some, it is more plausible that you can't complete an infinite number of journeys than that you must. . . . To others it is more plausible that you must complete an infinite number of journeys than that you can't.[13]

This has not proven to be easy. Benacerraf writes, "there is far from universal agreement on just what was wrong with his arguments. The debate has lasted these several thousand years. Most

[13] James Thomson. *Tasks and Supertasks* in Salmon, *Zeno's Paradoxes*, (New York: The Bobbs-Merrill Company, Inc., 1970), 103-104.

likely, it will last several thousand more . . ."[14] "Over the years," writes Vilenkin, "judgements concerning Zeno's paradoxes have changed many times. Just when it seemed that they had been completely refuted, a careful analysis would show that, whatever level of knowledge, there remained something unexplained . . ."[15] This debate will be summarized below.

Proposed Solutions

Aristotle - "Infinite" Time

Aristotle offers the earliest solutions to Zeno's arguments. He disagrees with Zeno's idea of the infinite, a false assumption that he attacks by stating, "there are two senses in which length and time and generally anything continuous [non-discrete] are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities."[16] Thus, while time and space can be divided to infinity, neither is infinite in a quantitative sense. He gives the example of a line (AB). Whatever the length of AB, it is bounded on either end (A or B) and thus is finite by definition. Thus, if in fact there is some notion of infinity in length, then the same thing could be said of the time in which the length might be traversed. In this case, then, an "infinite" gap could be spanned because an "infinite" amount of time would be available to accomplish the task.

So the earliest challenger denied the second premise. But did he solve the problem? It seems that Aristotle may only have proven Parmenides' point: the universe is actually one (i.e indivisible), for if the universe were *actually* made up of discrete units then can it not *actually* be

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[14]Paul Benacerraf. *Tasks, Supertasks, and the Modern Eleatics* in Salmon, *Zeno's Paradoxes*, (New York: The Bobbs-Merrill Company, Inc., 1970), 103-104.

[15]Vilenkin, *Infinity*, 7.

[16]Aristotle, *Physics,* Book VI:2 233b (315).

divided infinitely? It is this very idea that Zeno denied and argued against using the geometrical paradox, which he then illustrated with those presented here.[17]

According to some, the problem could not be dealt with adequately until the rise of nineteenth century mathematics. After citing a series of advances in mathematical theory including calculus and transfinite numbers Salmon writes, “It was only with nineteenth-century developments in mathematics that the convergent series generated by ‘The Achilles’ and ‘The Dichotomy’ could be handled.”[18] Yet he goes on to state that even these were inadequate and that various semantical problems and a definitive link between reality and abstract math were still wanting. It seemed it might be up to the following century’s disputants to put the issue to rest.

Max Black - Self Contradiction

Black begins by dispelling what he considers to be the flawed arguments of Descartes and Whitehead who postulated, much like Aristotle, that in reality an infinite number of actions could be performed. The argument has to do with surpassing a “convergent geometrical series,” that is, the addition of descending distances such as 100 + 10 + 1 + 1/10 + 1/100 . . . which describes the rate at which Achilles is catching up to the tortoise. If Achilles keeps running, the distance between he and the tortoise becomes quite small. “It tells us,” he writes, “correctly, when and where Achilles and the tortoise will meet, *if* they meet; but it fails to show that Zeno was wrong

[17]Either units in a collection have no size at all (for if a unit is divisible into parts, each part can then be divided an infinite number of times and thus one would eventually arrive at a part that was infinitely small), or each unit is infinite in size (for only an infinite part cannot be divided). See G.E.L. Owen, *Zeno and the Mathematicians* in Salmon, *Zeno's Paradoxes*, (New York: The Bobbs-Merrill Company, Inc., 1970), 141-145.

[18]Salmon, 24-25.

in claiming they could *not* meet."[19] The problem, notes Black, is that no matter how small the steps become there are still an infinite number of them to take. Thus, Achilles is being asked to perform an infinite series of tasks. It is this premise that Black attacks, calling the very notion of an "infinite series of acts" self-contradictory.[20]

In order to show that this is the case, Black proposes several "infinity machines" that are designed to complete a "super-task" - a task with an infinite number of steps. He describes several machines: Alpha – a machine that counts an infinite collection of marbles, each time counting faster and faster until finished, and then Beta – which moves one marble to a tray that Alpha then moves back into its own tray until this sequence has been repeated an infinite number of times. Later Gamma, Delta, Epsilon, and Phi are called forth to do similar exercises. In each case the question is asked about the number or position of the marbles. And in each case contradictory answers are given, thus proving that super-tasks are impossible to complete. He summarizes the problem's relation to Zeno's paradox when he writes, "every series of acts is like counting in requiring the successive doing of things, each having a beginning and end in space or time. And this is all that was used or needed in our arguments."[21]

Black criticizes Aristotle for allowing Zeno to assert an infinite number of steps for Achilles to take. Instead, Black points out that in *physical reality* the steps are not truly infinite – no matter how small they are they remain a fixed, finite number. "The infinity of this series is

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[19]Max Black, *Achilles and the Tortoise* in Salmon, *Zeno's Paradoxes*, (New York: The Bobbs-Merrill Company, Inc., 1970), 69-70.

[20]Ibid., 72.

[21]Ibid., 79.

then a feature of one way in which we find it useful to *describe* the physical reality; to suppose that therefore Achilles has to *do* an infinite number of things would be as absurd as to suppose that because I can attach two numbers to an egg I must make some special effort to hold its halves together."[22]

So Black attempts a solution by denying that the first premise is an accurate description of reality. He makes a distinction between our ability to posit an infinite dividing of things and the actuality of this task being accomplished. In doing so he also strikes at the arguments of his predecessors. The question remains, however – what is the connection between our description of reality and physical reality? Are they the same? If not, then couldn't this very self-contradiction actually support Zeno's contentions?

J.O. Wisdom - Category Mistake

Wisdom does not so much denounce Black's argument as adds to it what he considers a necessary element to keep it from leaving open a loophole for Zeno's presupposition. The loophole Wisdom sees is that by simply showing one premise to be self-contradictory Black has not shown Zeno's conclusion to be false. "Indeed," he writes, "the self-contradiction even plays into Zeno's hands, in a way that was not open to him with the logic of his day, for from a self-contradictory premiss all inferences are valid."[23] In other words, Zeno's argument may still describe the actual state of affairs – that physical reality is an illusion.

Wisdom's goal, then, is to show that Zeno does not accurately describe physical reality. He does so by demonstrating that physical distance cannot be split up into an infinite number of

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[22]Ibid., 80. (emphasis in original)

[23]Wisdom, *Racecourse*, 83-84.

mathematical points.[24] He illustrates this by asking how one might mark the points along a racecourse. No matter how small each mark was, it would always be larger than a mathematical point (which has no size at all). Thus an actually infinite number of marks could never fit into an actually finite distance. If the size of the marks were reduced to zero then they would cease to exist in physical reality and thus would exit the realm of the physical to which the paradoxes are supposed to refer. Zeno's argument is a category mistake and is thus moot.

But Richard Taylor criticized Black's (and therefore Wisdom's) conclusion by pointing out a "self-inconsistent notion" in his super-tasks. Blacks' problem, according to Taylor, is that he introduces a last member to an infinite set which is, by definition, non-existent – there is no such thing as an "infinitieth number" to reach.[25] According to Taylor a super-task could be performed in the manner Black describes because what is required is the performing of *all* of the steps and not a *last* one. Taylor describes Achilles' race by saying that he travels the following distances: "1/2+ 1/4 + 1/8 + . . . , then since the sum of this series is 1, Achilles can finish the course."[26]

James Thomson - Invalid Equivocation of Applicable Terms

Yet another attempt to solve the dilemma is proposed by James Thomson who claims that the argument itself is invalid due to an equivocation in terms. Thomson does not find Black's argument entirely satisfying due to the work of Taylor and Watling who argued for the second

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[24]Ibid. 85-88.

[25]William Craig.*The Kalam Cosmological Argument* (Eugene, Wipf and Stock Publishers, 1979) 178.

[26]Ibid., 179.

premise. He does not, however, find their arguments sufficient either. "Luckily," he writes, "we need not take sides in this dispute. For the argument stated above is not valid."[27] Thomson makes a distinction between an infinite *task* and infinite *tasking*. Clearly, if a thing can be infinitely divided then its division can be done infinitely often. This is not, however, the same as stating that such an operation can *have been* done.

It might seem that this is not the case because "it certainly makes sense to speak of someone having performed a number of tasks. But infinite numbers are numbers; therefore it must it must make sense to speak of someone having performed an infinite number of tasks."[28] But this misses the point. What Thomson seeks to demonstrate is that simply because one can conceive of a super-task that does not entail that it might actually be completed, and indeed it cannot. This is illustrated by Thomson's lamp.

If one had a lamp with an on/off switch at its base and one were to press that button an infinite number of times would the lamp be on or off? The answer, Thomson argues, is that the lamp is both on and off which is a contradiction. The contradiction arises from the fact that for every odd number of "switchings" there is a corresponding even number to counteract it. For every "on" there is an "off." He says that the mathematical answer to this series is useless because in reality the lamp cannot be both on and off simultaneously.

In summary, Thomson wishes to show that while it is true that there are an infinite number of points between any distance's end points, the applicability of this concept to an actual physical distance is not what Zeno thinks it is. It is not, as he believes some have thought, the

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[27] Thomson. *Tasks and Super-Tasks*, 90.

[28] Ibid., 93.

same as the problem of a man running an infinite number of miles. "So it is just wrong to say that the concept of [an infinite set] has no application to 'physical reality' (which is I think what Black and Wisdom are saying). But on the other hand the implicit use of this concept in the first premiss of the Zenoesque argument *is* a misleading one, and this is just what the second premiss calls attention to."[29]

Thus, there is an equivocation between "completing an infinite number of tasks" in the two premises – one refers to the fact that there are an infinite number of conceptual points and the other refers to one's crossing of an infinite number of real distances. The former is a matter of analysis, the latter is one of actuality. Either definition of "completing an infinite number of tasks" is acceptable, but whichever one is chosen will invalidate one of the two premises.

Paul Benacerraf - Affirming the Possibility of Super-Tasks

Thomson, like Black, was not without his critics. Paul Benacerraf concluded that "whether the lamp is on or off at the first instant after the completion of the super-task is irrelevant to the possibility of the task's being performed."[30] Benacerraf thinks this is the case due to Thomson's confusion of the instant "infinity + 1" with "infinitieth" instant. Thus, Thomson's conclusion only applies to instants before the task is completed and not after, for "the limit of the series is not in the series." Thus, Thomson is "trying to deduce a conclusion about the state of affairs at an instant following a progression from information pertaining only to states of affairs within the progression."[31]

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[29]Ibid., 101.

[30]Craig, *Kalam*, 179.

[31]Ibid.

Benacerraf further attacked Black's infinity machines by pointing out that should the distance between the trays grow smaller along with the other actions growing faster that eventually they would touch and the marble would be between both trays yet touching each. In the end, both Black and Thomson admitted defeat.[32] But were they truly defeated?

William Lane Craig - Re-Affirming the Impossibility of Super-Tasks

Craig notes at the outset of his defense of the impossibility of infinity machines that "it always seems dangerous to attempt to resuscitate an argument pronounced dead by even its original exponents, but in this case the risk seems to be worth it."[33] Indeed, the failure of Black and Thomson to prove their point in the manner they set out to do so does not negate the possibility of their conclusion remaining correct. Craig states that the real reason an infinity machine is impossible is that a super-task requires the completion of an "infinitieth" task which is absurd.[34]

The objections brought by both Benacerraf, and later, Grünbaum, are true, says Craig, in the mathematical realm, but not in the real world. The mistake is a common one according to Craig, and it involves a confusion between *logical* relations (which exist in the mathematical realm) and *causal* relations (which exist in the real world). "So while it is true that in the realm of abstract mathematical entities w+1 is not determined by an ' wth ' moment, it must be so in the real world of things."[35] In other words, while in the series 1, 2, 3 . . . the number 1 does not cause

[32]Ibid., 180.

[33]Ibid.

[34]Ibid.

[35]Ibid., 181.

the number 2, in the real world of, say, lamp switching, the state of the lamp at position 1 will cause the state of the lamp at position 2.

Mathematical examples are irrelevant to Thomson's case here because real, physical events are not related in the way that abstract mathematical objects are related. It is one thing to write ". . ." at the end of a series to denote infinity, but in the real world this is precisely where the argument fails.[36]

Craig further notes regarding super-tasks that the idea that an infinity machine could "complete all the tasks without performing a last task is nonsensical."[37] To have completed a series of actions is to complete all of them including the last one. Craig's point is that in reality there is no such thing as an "infinitieth" number – for by definition that number would be the unlimited series' limit which is a self-contradiction. Oddly, this solution is not entirely new. In fact it is based on distinctions made in a solution that was offered by the very first respondent to Zeno – only in another section of his writings than his original answer, which he admitted was lacking. Perhaps, as will be investigated below, the matter boils down to the true nature of infinity itself.

Aristotle - The Nature of the Infinite

In the first solution covered in this paper it was pointed out that Aristotle posited an "infinite" gap that could be spanned given an "infinite" amount of time (if by "infinite" was meant "infinitely divisible"). This, he later admitted, was not an adequate answer due to the

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[36]Wisdom, *Achilles*, 88. Craig paraphrased it succinctly: "The argument fails when it comes to the dots."

[37]Craig, *Kalam*, 182.

actual nature of distances and time. He then proposed that the problem be understood in the light of two important distinctions: “Therefore to the question whether it is possible to pass through an infinite number of units either of time or of distance we must reply that in a sense it is and in a sense it is not. If the units are actual, it is not possible: if they are potential, it is possible.”[38]

These two terms must be distinguished before this matter can be investigated fruitfully: (1) Actual Infinite (that which is infinite), and (2) Potential Infinite (that which is becoming infinite).[39] “An actual infinite is a set which contains an infinite number of members, as for example the set of all positive integers; {1, 2, 3, . . .}.”[40] Dedekin defined an actual infinite this way: “a system is said to be infinite if a part of that system can be put into one-to-one correspondence with the whole.”[41] By way of example, this would be to say that {1, 2, 3, . . .} = {2, 4, 6, . . . } in quantity. Leading transfinite mathematician George Cantor labeled the actual infinite as a_0 (“aleph null”). The potential infinite, on the other hand, is an ever-increasing set formed by successive addition – or an “inexhaustible finitude.”[42] The chief difference between the two is that only the potential infinite has real existence, for an actual infinite number of things cannot exist. This is because if one had an actual infinite number of things one more could always be added. But this is the definition of a potential infinite – commonly labeled ∞.

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[38]Aristotle, *Physics,* Book VI:2, 263a (*Great Books,* 349-350).

[39]Vilenkin, *Infinity,* 9.

[40]Richard Howe, "An Analysis of Williams Lane Craig's Kalam Cosmological Argument" (B.A., University of Mississippi, 1990), 8.

[41]Craig, *Kalam*, 67.

[42]Ibid., 9.

So far the possibility of the existence of an actual infinite has been denied but not proven impossible. William Lane Craig uses several illustrations to show why this is the case. He assumes the possibility of the existence of an actual infinite and then shows through *reductio ad absurdum* how this cannot be the case. One example, that of the infinite library, comes from the above-mentioned definition by Dedekin wherein {1, 2, 3, . . .} = {2, 4, 6, . . . }.[43] Suppose it were the case that one had an actually infinite number of books. Suppose further that the odd numbered books were black and the evens red. If one were to count them (which would, by the way, take an infinite amount of time) they would find that there were as many red books as black books. Oddly, though, if one were to remove the red books the collection would not decrease in size, for there would still exist an infinite number of black books (numbers 2, 4, 6, . . .). If the red books were placed back on the shelves the quantity would not increase, for there would still be an infinite number of books (numbers 1, 2, 3, . . .). What is even more bizarre is that the number of red books would be equal to the number of black books *plus* the number of red books {1, 3, 5, . . . } + {2, 4, 6, . . . } = {1, 2, 3, . . . }. The paradoxes do not stop there. If one were to check out all books above number 2 (numbers 3, 4, 5, . . .) then there would only be two books left (numbers 1 and 2) – yet an infinite number of books had been removed ({3, 4, 5, . . . } - {1, 2, 3, . . . }). So in this case $a_0 - a_0 = 2$! This is patently absurd.

Now all of this is not a problem for mathematics which only deals in abstraction. For example a triangle can be defined abstractly as a geometric figure with three sides, but this does not mean that an actual triangle exists anywhere - only that if one did this is what it would be. In

[43]Ibid., 82-86.

the same way, mathematicians can use things like a_0 to perform transfinite equations (most quite unimaginable, such as: $a_0^{a_0}$), but this does not mean that these figures represent something having actual existence. Simply because something can be defined it does not follow that it exists (such as: "the 15th planet in our solar system"), or that it *can* exist (such as a square circle). One can easily perform mathematical equations using unreal things (like 5 unicorns - 4 unicorns = 1 unicorn), for the abstract only tells what *would* be true *if* that which is being represented actually existed. Infinity theorists realize this as well. "Some of the most eager enthusiasts of the system of transfinite mathematics are only too eager to agree that these theories have no relation to the real world."[44] Several notable mathematicians have given other proofs that show that an actual infinite results in impossible situations, these include: Burali-Forti's antimony, Cantor's antinomy, and Russell's antimony. Each of these results in a contradiction if instantiated into the real world.[45]

Due to the fact that an actually existing infinite number of things entails contradictory absurdities both within and without mathematics it cannot be the case that an actually infinite number of things can exist. This being the case, Zeno has committed the fallacy of equivocation concerning the term "infinite" by confusing a concrete example (a potential infinite) with a mathematical abstract (an actual infinite):

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[44]Ibid., 87.

[45]Ibid., 90. Also see Kant's critique in Stephen Barker, *Philosophy of Mathematics* (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1964), 71-72.

(1) In order for movement to take place an [*potential*] infinite number of points must be traversed. (TRUE)
(2) An [*actual*] infinite number of points cannot be traversed. (TRUE)
(3) Therefore movement cannot take place. (FALSE)

Zeno's Failure?

While it may be said that Zeno's paradoxes have been answered by the distinction between physical reality and abstract ideas an interesting question arises: Did Zeno succeed in his task? From what is said of him by Plato and Aristotle it seems that he was defending Parmenides' Monism against a particular system, that of the Pythagoreans. Now, the Pythagoreans were not content with simply numbering all things in the universe (which, by definition means that there were multiple things in the universe), they asserted that things *are* numbers.[46] Furthermore, they regarded these numbers as being spatially existent – not mere abstractions or descriptions.[47] The difficulty here is that if this is the case then one might very well conclude that physical reality really can be divided infinitely, for real numbers can all be halved, and those halves halved, etc. If real numbers correspond to real units of material reality then Zeno has made no equivocation and the paradoxes hold. Thus, in a sense, Zeno succeeded in defeating his master's opponents although he ultimately failed to uphold his master's conclusion.

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[46]Copleston, *Philosophy*, 33.

[47]Ibid., 34. (i.e. "One" is a point, "two" a line, "three" a surface, "four" a solid.)

Conclusion

Zeno's importance should not be discounted whether his paradoxes have been answered or not. "Zeno's work did not consist of making additions to the Eleatic metaphysics, but of developing new operational methods for dialectic. Zeno, thus, gave a gigantic impetus to the development of philosophy. . . . in the mathematical character of Zeno's abstractions lies the last remnant of the fixation to spatial conceptions."[48] This set the stage for the next great phase of philosophical speculation. For using these methods, Plato would arrive at the first truly transcendental metaphysic.[49] Further, Zeno has been credited with the development of infinite process theories that eventuated in the discovery of the calculus by Newton and Leibniz.[50]

While some may view Zeno's paradoxes as mere sophistry with no more value than to produce skepticism, it is astonishing that such puzzles, thought up by a pre-scientific (and even pre-socratic) thinker could remain relevant almost 2,500 years after their creation. Further, with the stakes as high they are regarding his conclusions regarding our perception of reality, we dare not take them lightly.

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[48]Sinnige, *Matter and Infinity*, 109.

[49]Ibid.

[50]Keith Devlin, *Mathematics - The Science of Patterns* (New York: Scientific American Library, 1997), 74-78.

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